

Study of India VIX options pricing using Black-Scholes model

Arun Chauhan¹
Ravi Gor²

Abstract

India VIX is the first volatility index in India based on the order book of NIFTY Options. For this, the best bid-ask quotes of near and next-month NIFTY options contracts which are traded on the F&O segment of NSE are used. In February 2014, NSE introduced Futures contracts based on India VIX. There are strong possibilities that options on India VIX may be traded in NSE in future. In this paper, we compute the India VIX

option prices using Black-Scholes model and Generalized Black-Scholes model. Then we make some observations by comparing these two prices of India VIX options.

Keywords: *Futures and Options, NIFTY, India VIX, Black-Scholes model, Generalized Black-Scholes model*

¹ Department of Mathematics, J & J College of Science, Nadiad

² Department of Mathematics, Gujarat University, Ahmedabad

Introduction

Derivatives have become the most important financial instruments. A derivative is a type of contract between two persons/parties, which derives its value from an underlying asset like stocks, currencies, commodities, bonds, interest rates, etc. The most common derivatives are options, forwards, futures, and swaps.

Among all different types of derivatives, options are the most common derivatives traded all over the world. Options are a type of derivative contract which gives the buyer/holder a right to buy or sell the underlying asset at the agreed price (strike price) during a specified period (expiration time). An option contract gives buyers the right to exercise options; the buyer is not under any obligation to exercise the option. The option seller is known as the option writer. American options can be exercised at any time before the expiry of the option contract; on the other hand, European options can be exercised only on the date of expiration. In a very broad sense, there are mainly two types of options – Call option and Put option. Call options are contracts that give the buyer the right, but not obligation, to buy the underlying asset in the future at the strike price. You would purchase a call option if you believed that the price of the underlying asset/stock was likely to increase over a specified period. Put options are the opposite of calls. The owner of a put has the right, but not obligation, to sell the underlying asset in the future at the strike price. Therefore, you would buy a put option if you were expecting the price of the underlying asset/stock to fall.

Options can further be classified into 'Exchange-traded options' and 'Over the counter' options. Exchange-traded options, generally known as listed options, are the most common form of options. The term 'Exchange Traded' is used to describe an options contract that is listed on a public trading exchange. In India, BSE and NSE are the main exchange-traded markets. Over the Counter (OTC) options can only be traded on the OTC markets. So, it is less accessible to the general public. The securities traded on OTC are

mainly of companies not listed on formal exchanges. There are many customized contracts that are traded on OTC like Exotic option contracts.

When we use the term options, we are generally referring to stock options, where the underlying asset is shares in a publicly listed company. While these are certainly very common, there are also several other types of options in which the underlying security is something else. For example, Index Options are very similar to stock options, but rather than the underlying security being stocks in a specific company, it is an index – such as the NIFTY 50. Similarly, the underlying security of a Futures Options is a specified futures contract. A futures option provides the owner the right to enter into that specified futures contract.

Volatility is simply a measure of the degree of price movement in a stock, futures contract, etc. There are two types of volatilities – Historical volatility and Implied volatility.

Historical volatility is the volatility depending on the historical prices of stocks or assets. Normally, historical volatility is measured by taking the percentage change in daily closing prices of a stock and calculating the average over a given period of time. Then this average is expressed in annualized percentage.

Implied volatility is the current/market volatility of a stock, as estimated by its option price. An option's value depends on many components like the current stock price, strike price, expiration date, the implied volatility of the stock, dividends paid by the stock (if any), and risk-free interest rates. If we know the option price and all the above components, except volatility, then we can modify the option-pricing model to calculate the implied volatility of the particular stock.

Volatility Index is a measure of the market's expectation of volatility for the next few days. Usually, if the market moves quickly up or down, the volatility index rises. Volatility Index is different from the NIFTY index. The NIFTY index is computed using the price

movement of the underlying stocks. Volatility Index is computed by using the order book of the underlying index options. The volatility index is denoted as an annualized percentage of movement in the market for the next few days. India VIX is a volatility index traded and computed by NSE based on the order book of NIFTY Option contracts. For this, the best bid-ask quotes of near and next-month NIFTY options contracts that are traded on the F&O segment of NSE are used. India VIX indicates the investor's perception of the market's volatility in the near term. It predicts the market volatility over the next 30 calendar days. Higher the India VIX values, higher the expected volatility of the market, and vice versa.

The Chicago Board Options Exchange (CBOE) volatility index (VIX) likewise called the fear index, is a volatility index derived using S&P 500 index options. It is a well-known measure of the market's desire for volatility in the following 30 scheduled days. In 1993, CBOE began distributing real-time VIX information by utilizing the equation proposed by Whaley (1993). Consequently, on March 26, 2004, CBOE propelled VIX futures; VIX options were propelled on February 24, 2006. Since the VIX-based derivative gives effective risk hedging to investment in stocks/indices, the VIX futures and options are fluid in CBOE. The total volume exchanged of VXOs (VIX options) was 328,992,577 agreements during 2017, which is equivalent to 654,061 agreements exchanged every day. Today, the VIX has become the standard gauge of investor fear and sentiment in the financial market, which attracts the attention of the international financial markets. Accordingly, NSE launched the India VIX in April 2008. India VIX is calculated by the same formula as the CBOE with some suitable amendments. In February 2014, NSE introduced a Futures contract based on India VIX. But unfortunately, this contract did not become popular among market participants and today, no futures contract is traded in the market. So, there are strong possibilities that options on India VIX may be traded on NSE in future.

In this paper, we will answer the following questions:

1. By which method can we find the theoretical price of India VIX options?
2. Can Black-Scholes option pricing model help us to compute the price of India VIX options?
3. Are there any other models by which we can calculate and compare the India VIX options price?

Literature Review

Black and Scholes (1973) first derived a theoretical valuation formula for options. This formula is also applicable to corporate liabilities such as common stock, corporate bonds, and warrants. In particular, the formula can be used to derive the discount that should be applied to corporate bonds. Under a similar assumption, Black (1976) derived the pricing formula for futures options.

There are many modified B-S models in literature in which log-returns of underlying assets are not taken as normally distributed. In particular, generalized beta distribution of the second kind was used by McDonald and Bookstaber (1991) while Burr-3 distribution was adopted by Sherrick (1996). Other examples include Weibull distribution used by Savickas (2002), g-and-h distribution studied by Dutta and Babbel (2005) and generalized gamma distribution adopted by Fabozzi (2009). Singh and Gor (2020a) compared the B-S option pricing model and the new option pricing model in which underlying stock returns follow the 'gumbel distribution' at maturity and analysed the result of comparison on actual market data. Singh and Gor (2020b) also compared B-S model and a model in which stock returns follow 'truncated gumbel distribution'.

A modification to the B-S formula considers that option traders often have their own expected (finite) range of the underlying price in mind, which is a very reasonable and attractive idea to refine B-S model. S. P. Zhu (2017) made such modifications and assumed that the log-returns of the underlying asset follow a truncated normal distribution within a certain period with fixed upper and lower bounds. Chauhan and Gor

(2020a) have compared B-S model with modified B-S model in which underlying stock returns follow truncated normal distribution on option prices of selected Indian IT stock options listed on NSE.

Kalra (2015) analysed the effects on volatility due to the different types of situations like, economic slowdown, foreign investments in Indian stock market, etc. Wats (2017) examined the effect of expiration periods of derivatives on the volatility of Indian spot market. She also observed that due to introduction of options & futures with the near months, the volatility of the spot market has increased in expiration day as well as expiration week. Singh (2015) made an attempt to forecast the volatility of NIFTY 50 stock index returns. He analysed that among many volatility forecasting models, ARIMA (1,0,1) model performs best for forecasting the volatility of NIFTY 50 index returns.

Whaley (1993) showed how volatility derivatives can be used to provide a simple, cost-effective means for hedging the market volatility risk of portfolios that contain options or securities with option-like features. He also provided details about how market volatility derivatives should be valuable risk management tools for options market makers.

Hung-His Huang (2019) reasonably evaluated VXO (volatility index option) prices on the Taiwan stock index using four popular pricing models, namely, Black-Scholes (BS), square root (SQR), lognormal Ornstein-Uhlenbeck (LOU), and GARCH models.

CBOE VIX white paper provides detailed knowledge about how to calculate Volatility index from S&P 500 index options order book.

Chauhan and Gor (2020b) have explained the process of calculating the theoretical price of India VIX options by using the classical B-S model. In this paper, we have extended this process and compared the B-S price of India VIX options with options price computed by Generalized B-S model.

A white paper on India VIX explains the computational methodology of India VIX volatility index derived from order book of NIFTY 50 index options traded on NSE.

Computation Methodology of India VIX

India VIX uses the computation methodology of CBOE, with suitable amendments to adapt to the NIFTY options order book.

The formula for the India VIX calculation is:

$$\sigma^2 = \frac{2}{T} \sum \frac{\Delta K_i}{K_i^2} e^{RT} Q(K_i) - \frac{1}{T} \left[\frac{F}{K_0} - 1 \right]^2$$

σ = India VIX/100

T = Time to expiration

K_i = Strike price of i^{th} out of the money option

ΔK_i = Interval between strike prices

R = Risk-free interest rate to expiration

$Q(K_i)$ = Midpoint of the bid ask quote for each option contract with strike K_i

F = Forward index taken as the latest available price of NIFTY futures contract of corresponding expiry

K_0 = First strike below the forward index level, F

Black-Scholes (BS) model

Let S_0 be the current price of stock, V be the value of European call options on this stock with,

Let S_0 be the current price of stock, V be the value of European call options on this stock with,

X = Strike price

T = time to expiration

σ = volatility of stock (constant)

r = risk-free interest rate

Then the value, V of call today is given by,

$$V = S_0 N(d_1) - X e^{-rT} N(d_2)$$

In this formula, $N(x)$ denotes the standard normal distribution function. That is,

$$N(x) = P[Z \leq x] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-\frac{t^2}{2}} dt$$

And,

$$d_1 = \frac{\ln\left(\frac{S_0}{X}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sqrt{T}\sigma} \text{ and } d_2 = d_1 - \sigma\sqrt{T}.$$

Maple code for BS model

```

> with(Statistics) :
> X := RandomVariable(Normal(0, 1))
X := _R
> g(y) := PDF(X, y);
g := y -> Statistics:-PDF(X, y)

> N(a) := \int_{-\infty}^a g(y) dy
N := a -> \int_{-\infty}^a g(y) dy

> DI(S, X, r, \tau, \sigma) := \frac{\left(\ln\left(\frac{S}{X}\right) + \left(r + \frac{\sigma^2}{2}\right)\tau\right)}{\sqrt{\tau}\sigma}
DI := (S, X, r, \tau, \sigma) -> \frac{\ln\left(\frac{S}{X}\right) + \left(r + \frac{1}{2}\sigma^2\right)\tau}{\sqrt{\tau}\sigma}

> D2(S, X, r, \tau, \sigma) := DI(S, X, r, \tau, \sigma) - \sqrt{\tau}\sigma
D2 := (S, X, r, \tau, \sigma) -> DI(S, X, r, \tau, \sigma) - \sqrt{\tau}\sigma

> V(S, X, r, \tau, \sigma) := S \cdot N(DI(S, X, r, \tau, \sigma)) - X \cdot e^{-r\tau} \cdot N(D2(S, X, r, \tau, \sigma))
V := (S, X, r, \tau, \sigma) -> SN(DI(S, X, r, \tau, \sigma)) - X e^{-r\tau} N(D2(S, X, r, \tau, \sigma))

> evalf(V(1976.5, 2000, 0.0625, 0.07945, 0.247))
48.5448031

```

Generalized Black-Scholes (GBS) model

In this section, first we define truncated normal distribution; after that, closed form option pricing formula by changing truncated normal distribution in Black-Scholes model is shown. This Modified Black-Scholes model and its closed form pricing formula were first derived by S.P.Zhu (2017).

Truncated normal distribution

If a random variable X is assumed to follow a truncated normal distribution with $X \in [a, b]$, then its probability density function is,

$$f(x; \mu, \sigma, a, b) = \begin{cases} \frac{\frac{1}{\sigma} \phi\left(\frac{x-\mu}{\sigma}\right)}{\Phi\left(\frac{b-\mu}{\sigma}\right) - \Phi\left(\frac{a-\mu}{\sigma}\right)}, & a \leq x \leq b \\ 0, & \text{otherwise} \end{cases} \quad (3.1)$$

Where $\phi(\cdot)$ and $\Phi(\cdot)$ represent a standard normal density function.

If we denote that $Y = \frac{S_t}{S_0}$, it is not difficult to find that the probability density for Y can be expressed as

$$\frac{1}{y} f(\ln y; \mu t, \sigma\sqrt{t}, a, b). \text{ So,}$$

$$f_Y(y) = \begin{cases} \frac{1}{y} \frac{\frac{1}{\sigma\sqrt{t}} \phi\left(\frac{\ln y - \mu t}{\sigma\sqrt{t}}\right)}{\Phi\left(\frac{b - \mu t}{\sigma\sqrt{t}}\right) - \Phi\left(\frac{a - \mu t}{\sigma\sqrt{t}}\right)}, & e^a \leq y \leq e^b \\ 0, & \text{otherwise} \end{cases} \quad (3.2)$$

A closed-form pricing formula

The formula for pricing European call options with truncated normally distributed underlying as given in [9] is,

$$V_c = S_0 \frac{\Phi\left(\frac{b - \mu t}{\sigma\sqrt{t}} - \sigma\sqrt{t}\right) - \Phi\left(\frac{\ln \frac{K}{S_0} - \mu t}{\sigma\sqrt{t}} - \sigma\sqrt{t}\right)}{\Phi\left(\frac{b - \mu t}{\sigma\sqrt{t}} - \sigma\sqrt{t}\right) - \Phi\left(\frac{a - \mu t}{\sigma\sqrt{t}} - \sigma\sqrt{t}\right)} - K e^{-rt} \frac{\Phi\left(\frac{b - \mu t}{\sigma\sqrt{t}}\right) - \Phi\left(\frac{\ln \frac{K}{S_0} - \mu t}{\sigma\sqrt{t}}\right)}{\Phi\left(\frac{b - \mu t}{\sigma\sqrt{t}}\right) - \Phi\left(\frac{a - \mu t}{\sigma\sqrt{t}}\right)}$$

Maple code for GBS model

> with(Statistics) :

> X := RandomVariable(Normal(0, 1))

X := _R

> g(y) := PDF(X, y);

g := y → Statistics:-PDF(X, y)

> N(a) := $\int_{-\infty}^a g(y) dy$

N := a → $\int_{-\infty}^a g(y) dy$

> T(S, K, r, t, a, b) :=

$$S \cdot \frac{N\left(\frac{b}{\sqrt{t}} - \sqrt{t}\right) - N\left(\frac{\ln\left(\frac{K}{S}\right)}{\sqrt{t}} - \sqrt{t}\right)}{N\left(\frac{b}{\sqrt{t}} - \sqrt{t}\right) - N\left(\frac{a}{\sqrt{t}} - \sqrt{t}\right)} - K \cdot e^{-rt} \cdot \frac{N\left(\frac{b}{\sqrt{t}}\right) - N\left(\frac{\ln\left(\frac{K}{S}\right)}{\sqrt{t}}\right)}{N\left(\frac{b}{\sqrt{t}}\right) - N\left(\frac{a}{\sqrt{t}}\right)}$$

> evalf(T(60, 66, 0.06, 0.166, -1, 1))

7.12459159

Where, $T(S, K, r, t, a, b)$ is the price of European call for given data.

S : Stock price

K : Strike price

r : riskless interest rate

T : time to maturity

a : lower bound for truncated distribution

b : upper bound for truncated distribution

Computation of India VIX Option prices

We used the following terms in the calculation table.

- Observed date: A date when market price of particular options contract was taken.
- Spot price: Market price of underlying at time t=0.
- Strike price: A fixed price of an underlying agreed at the time of contract.
- Rate of interest: MIBOR rate at corresponding date.
- Time to Maturity: Time remaining in years for expiration of contract.
- Volatility: A rate at which the price of underlying increases or decreases for a given set of returns.
- BS price: Value of India VIX options computed by using Black-Scholes model
- GBS price: Value of India VIX options computed by using Generalized Black-Scholes model

For calculation, we select closing price of India VIX on the date of 09th August 2019, which is 15.8450. We compute value of European call options for different strike prices. The historical data of India VIX volatility index have been collected from the NSE website. Annualized volatility is based on last 30 days India VIX closing prices. We take seven different strike prices and seven different expiration dates for calculation. For finding option prices by GBS model, first we fix lower bound a= -0.1 and upper bound b=0.1 and then compare the prices computed by two models, BS model price and GBS model. Price of options are calculated by Maple software.

Table 1: For the Strike price K=10

Observed date	Spot price	Strike price	Rate of interest	Time to maturity	Volatility	BS price	GBS price
09/08/2019	15.845	10	0.054	0.013699	0.8812	5.853559	6.743148
09/08/2019	15.845	10	0.054	0.032877	0.8812	5.866637	7.596083
09/08/2019	15.845	10	0.054	0.052055	0.8812	5.886547	8.07171
09/08/2019	15.845	10	0.054	0.071233	0.8812	5.91602	8.380613
09/08/2019	15.845	10	0.054	0.109589	0.8812	5.997392	8.770701
09/08/2019	15.845	10	0.054	0.205479	0.8812	6.25812	9.282207
09/08/2019	15.845	10	0.054	0.320548	0.8812	6.588487	9.644137

Table 2: For the Strike price K=11.5

Observed date	Spot price	Strike price	Rate of interest	Time to maturity	Volatility	BS price	GBS price
09/08/2019	15.845	11.5	0.054	0.013699	0.8812	4.355192	4.87203
09/08/2019	15.845	11.5	0.054	0.032877	0.8812	4.385955	5.279284
09/08/2019	15.845	11.5	0.054	0.052055	0.8812	4.443579	5.474053
09/08/2019	15.845	11.5	0.054	0.071233	0.8812	4.516412	5.59497
09/08/2019	15.845	11.5	0.054	0.109589	0.8812	4.679384	5.751285
09/08/2019	15.845	11.5	0.054	0.205479	0.8812	5.090308	5.990629
09/08/2019	15.845	11.5	0.054	0.320548	0.8812	5.537253	6.20417

Table 3: For the Strike price K=13

Observed date	Spot price	Strike price	Rate of interest	Time to maturity	Volatility	BS price	GBS price
09/08/2019	15.845	13	0.054	0.013699	0.8812	2.871347	3.020692
09/08/2019	15.845	13	0.054	0.032877	0.8812	2.986961	3.116136
09/08/2019	15.845	13	0.054	0.052055	0.8812	3.12623	3.164011
09/08/2019	15.845	13	0.054	0.071233	0.8812	3.264737	3.199544
09/08/2019	15.845	13	0.054	0.109589	0.8812	3.524414	3.258131
09/08/2019	15.845	13	0.054	0.205479	0.8812	4.078761	3.384355
09/08/2019	15.845	13	0.054	0.320548	0.8812	4.62487	3.526145

Table 4: For the Strike price K=15.5

Observed date	Spot price	Strike price	Rate of interest	Time to maturity	Volatility	BS price	GBS price
09/08/2019	15.845	15.5	0.054	0.013699	0.8812	0.839189	0.565639
09/08/2019	15.845	15.5	0.054	0.032877	0.8812	1.196602	0.592436
09/08/2019	15.845	15.5	0.054	0.052055	0.8812	1.460443	0.607945
09/08/2019	15.845	15.5	0.054	0.071233	0.8812	1.680031	0.621194
09/08/2019	15.845	15.5	0.054	0.109589	0.8812	2.045538	0.645605
09/08/2019	15.845	15.5	0.054	0.205479	0.8812	2.745997	0.70335
09/08/2019	15.845	15.5	0.054	0.320548	0.8812	3.3939	0.770917

Table 5: For the Strike price K=17

Observed date	Spot price	Strike price	Rate of interest	Time to maturity	Volatility	BS price	GBS price
09/08/2019	15.845	17	0.054	0.013699	0.8812	0.252432	0.028492
09/08/2019	15.845	17	0.054	0.032877	0.8812	0.578312	0.035298
09/08/2019	15.845	17	0.054	0.052055	0.8812	0.835068	0.039367
09/08/2019	15.845	17	0.054	0.071233	0.8812	1.053574	0.042871
09/08/2019	15.845	17	0.054	0.109589	0.8812	1.422705	0.049352
09/08/2019	15.845	17	0.054	0.205479	0.8812	2.139888	0.064721
09/08/2019	15.845	17	0.054	0.320548	0.8812	2.808899	0.082718

Table 6: For the Strike price K=17

Observed date	Spot price	Strike price	Rate of interest	Time to maturity	Volatility	BS price	GBS price
09/08/2019	15.845	18.5	0.054	0.013699	0.8812	0.052419	0.083903
09/08/2019	15.845	18.5	0.054	0.032877	0.8812	0.246587	0.108731
09/08/2019	15.845	18.5	0.054	0.052055	0.8812	0.446082	0.112506
09/08/2019	15.845	18.5	0.054	0.071233	0.8812	0.632509	0.111263
09/08/2019	15.845	18.5	0.054	0.109589	0.8812	0.967645	0.103905
09/08/2019	15.845	18.5	0.054	0.205479	0.8812	1.656714	0.078012
09/08/2019	15.845	18.5	0.054	0.320548	0.8812	2.321494	0.043852

Table 7: For the Strike price K=20

Observed date	Spot price	Strike price	Rate of interest	Time to maturity	Volatility	BS price	GBS price
09/08/2019	15.845	20	0.054	0.013699	0.8812	0.007771	0.384166
09/08/2019	15.845	20	0.054	0.032877	0.8812	0.094145	0.591063
09/08/2019	15.845	20	0.054	0.052055	0.8812	0.224697	0.658328
09/08/2019	15.845	20	0.054	0.071233	0.8812	0.365927	0.68555
09/08/2019	15.845	20	0.054	0.109589	0.8812	0.646155	0.697917
09/08/2019	15.845	20	0.054	0.205479	0.8812	1.276418	0.65988
09/08/2019	15.845	20	0.054	0.320548	0.8812	1.917464	0.583355

Effect of bounds on option prices

In this section, we highlight the relation between BS model price and GBS model price using graphs by changing the bounds used in truncated normal distribution. For seven different strike prices, we compare prices by changing bounds. VIX price is 15.845, interest rate is 0.054, time to maturity is 0.01369 and volatility is 0.8812.

For each fixed strike price, we take different bounds as described in the following table:

a	b
-0.02	0.02
-0.08	0.08
-0.14	0.14
-0.2	0.2
-0.26	0.26
-0.32	0.32
-0.38	0.38
-0.44	0.44
-0.5	0.5

In the following graphs, the blue line shows BS price and red line shows GBS price; on x-axis, we take bounds, and on y-axis, we take option prices.

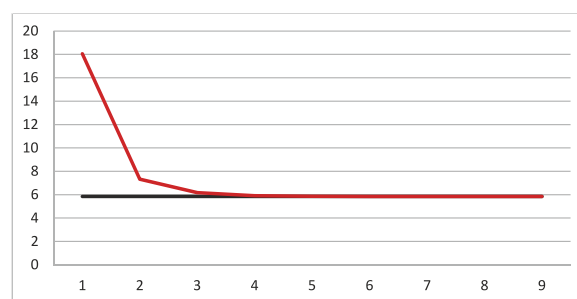


Fig.1 For the Strike price K=10

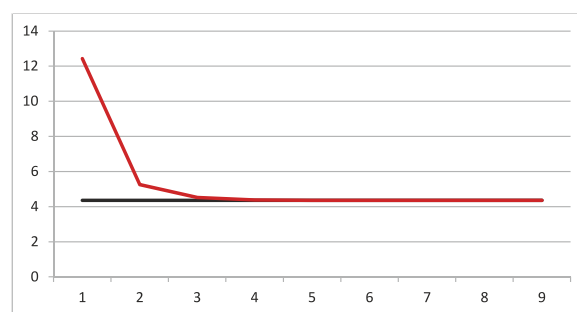


Fig.2 For the Strike price K=11.5

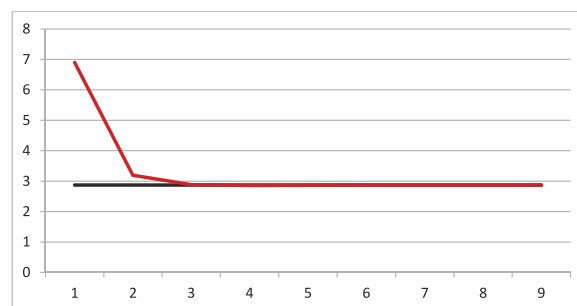


Fig.3 For the Strike price K=13

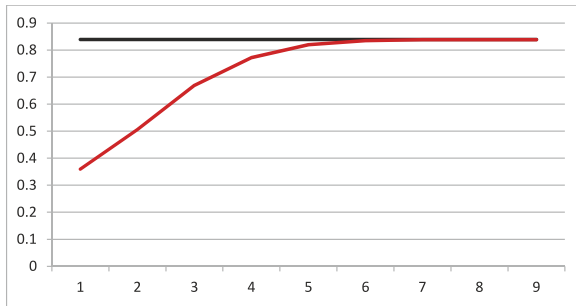


Fig.4 For the Strike price K=15.5

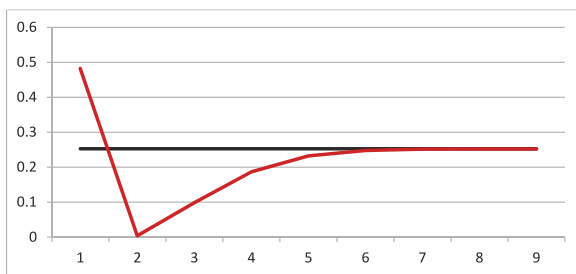


Fig.5 For strike price K=17

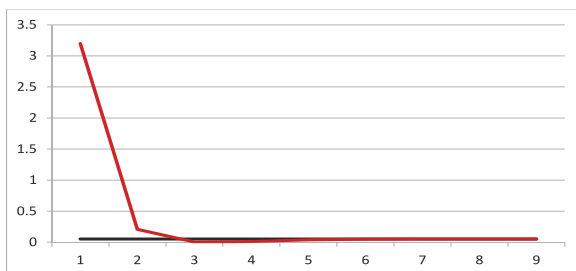


Fig.6 For strike price K=18.5

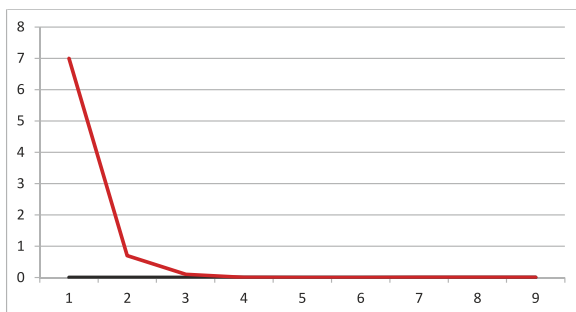


Fig.7 For strike price K=20

Observations

In Section 8, we calculate option prices by using both BS and GBS models. Then in Section 9, we fix all the components used in BS and GBS models for pricing options and only change upper and lower bounds used in truncated normal distribution in GBS model. After changing different bounds, we compare GBS price with BS price by graphs for different strike prices. By observing all the graphs, we can clearly say that as bounds of truncated normal distribution increases, the GBS model price converges to BS model price.

Applicability and Generalizability

As we all know, CBOE introduced options on VIX in 2006 to give investors more direct access to trade in volatility. Direct trading in volatility provides significant opportunities of hedging and speculation for traders. The main benefit of VIX options is its usefulness in portfolio diversification. In India, we have India VIX volatility index. But we don't have options on India VIX index yet. If we introduce options on India VIX, it can be beneficial as a new tool for Indian investors for hedging as well as speculating. So, in this paper, we have analysed two option pricing models which may be useful to find theoretical price of India VIX options if introduced in the Indian stock market.

Conclusions

In this study, we describe the computational methodology for pricing options on India VIX volatility index. Since options on India VIX are not introduced by NSE yet, we calculate option prices for hypothetical strike prices, expiration dates and bounds. We select market price of India VIX and compute the option price by Black-Scholes options pricing model and Generalized Black-Scholes model for seven different strike prices. This study may be useful when NSE will introduce options on India VIX.

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Arun Chauhan received the M.Sc. and M.Phil. degree in Mathematics from Sardar Patel University, Vallabh Vidyanagar, Gujarat. He is Assistant Professor of Mathematics in J. & J. College of Science, Nadiad, Gujarat. He has more than six years of teaching experience at the undergraduate level. He is also a PhD candidate in the Department of Mathematics, Gujarat University under the supervision of Dr. Ravi Gor. His research interests include Financial Mathematics, Option pricing models and Volatility of Financial Markets. He can be reached at arunchauhanfmg@gujaratuniversity.ac.in

Ravi Gor is a PhD in Mathematics from Gujarat University and PDF of the Department of Business Administration, University of New Brunswick, Canada. He is currently working at Department of Mathematics, Gujarat University and is Co-ordinator of the Five-Year Integrated M.Sc. Programs in AIML, Data Science and Actuarial Science. His broad research interests are in operations research, operations - Finance - Marketing interface, Industrial Engineering, Operations Management and Supply Chain Management. He has published 12 books and has about 70 publications in refereed national and international journals. He can be reached at ravigor@gujaratuniversity.ac.in